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DESIGN OF A BI-DIRECTIONAL SERIES RESONANT CONVERTER

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ABSTRACT

This paper will develop the circuit theory using Laplace transform techniques for a bi-directional series resonant converter with a shunt loaded inductor. The analysis will form the basis of a bi-directional series resonant converter design that can be fabricated and tested in the future. The proposed converter, because of its zero switching capability when built with advance switches like the MOS-controlled thyristor, can achieve efficiencies approaching 99%. The topology is capable of boost-buck operation without a transformer. Bi-directional power control will be extremely important to a wide variety of areas including electric vehicles, utility power, aircraft power, and shipboard power. Specific applications for bi-directional series resonant inverters include industrial motor controller, voltage regulator for hybrid-electric vehicle engine/generator, motor controller for aircraft actuators or electric vehicle propulsion, and inverter/rectifier for batteries and static Var controller.

INTRODUCTION

Many electrical and electromechanical applications require bi-directional power flow. The largest commercial application of bi-directional power control today is industrial motor control. Another common application is in uninterruptable power supplies for computers and emergency service systems. Similar to the uninterruptable power supply, battery charging and output regulation can be performed by a single bi-directional converter. Satellites often today require a separate battery charger to produce efficient charging from photovoltaic source and a parallel voltage regulator to provide constant bus voltage. Static Var controllers, used by modern utility systems to improve power quality, contain capacitors and a bi-directional converter to vary the effective capacitance in the power system.

A bi-directional converter based on a resonant inverter circuit offers enormous advantages over conventional square wave or pulse width modulated converters currently used for equivalent applications. The resonant circuit permits switching near the zero voltage or zero current points of the resonant waveform. Zero current switching virtually eliminates energy loss from the stored energy in the magnetic field caused by stopping the inductor current instantly. Zero voltage switching virtually eliminates energy loss from the stored energy in the circuit capacitors. Also, a resonant circuit, when designed with high voltage gain, can eliminate the need for a transformer.

BACKGROUND OF BIDIRECTIONAL CONVERTERS

The best illustration of the need for a bi-directional converter is in a motor controller application. Motor controllers usually perform four basic functions: starting and stopping the motor, rotating the motor in a forward and reverse direction, controlling the speed, and protecting the machine from damage. The speed of a machine is approximately controlled by the input voltage and the torque is approximately controlled by the current. Starting a motor requires a large amounts of torque at low speed, that translates to large current at low voltage. Stopping a motor requires that the kinetic energy from the high rotational speed must be removed initially with low torque. The rotor speed remains in the same direction because of mechanical inertia, so an opposite torque created by a reverse current flow is needed to de-energize or regeneratively brake the shaft. After the shaft has stopped, the input voltage can be switched, changing the speed of the motor in the reverse direction. Finally, the motor can be braked in the reverse direction permitting complete speed and direction control of the motor. This capability is called four quadrant operation, because when the torque is plotted versus the speed of a motor, the resulting four quadrants of the graph represent the four operations of the motor. A motor that only requires starting and stopping without reversing capability operates in the two quadrant mode.

The three general categories of converters capable of bi-directional power flow are phase controlled cycloconverters, current fed inverters, and voltage fed inverters. Cycloconverters change the frequency of the source to a more useful frequency for the load. Controlling the electrical frequency of a motor controls the speed of the motor. Cycloconverters were first developed in

1930's to control electric locomotive traction drive. The advent of mercury arc rectifiers made the technique possible. However the early schemes were technically difficult and expensive to build. The invention of solid state thyristors and other semiconductor switches in the 1950's and 1960's revived the concept of controlling electrical machinery with electronic control. The development of microprocessor integrated circuits in 1970's simplified the development of sophisticated control systems for converters. These developments also permitted the development of the current fed and voltage fed inverters.

Cycloconverters are strictly limited to coupling AC sources to AC loads and provide voltage regulation of the output through simple phase control of the switches. This limitation means cycloconverters are notoriously inefficient when switching an inductive load. Inverters by definition convert DC voltages to AC waveforms and, if a rectifier stage is place at the DC side, can also convert AC inputs. The output voltage or current can be regulated by square wave modulation or pulse width modulation to mimic the natural sine waveform as closely as possible. In addition, a resonant circuit can be added to produce an actual sinusoidal waveshape if desired. The resonant circuit allows the converter to switch at very low or zero voltages and currents (soft switching), reducing the problems caused by switching at full voltage or current (hard switching).

Resonant inverters are categorized as parallel resonant and series resonant, where the former is a parallel combination of an inductor and capacitor and the latter is the series combination of the two. Parallel resonant inverters use a square current wave created by switching whereby series resonant inverters use a square voltage wave. A parallel resonant inverter requires a constant current source, while a series resonant inverter prefers a constant voltage source. Since most power sources like generators and batteries tend to produce constant voltages, series resonant inverters are more suitable for many applications. Additionally, series circuits are more fault tolerant because the inductor naturally limits any fault current.

Several workers in the field of high power converters have proposed the resonant based converters as excellent candidates for high power bi-directional applications. Schwarz (1978) demonstrated a four quadrant AC/AC and AC/DC series resonant converter with a series connected load. Klaassens (1989) performed a steady state analysis of the bi-directional series resonant converter using state space analysis techniques. Klaassens demonstrated that a four quadrant series resonant converter is capable of limited voltage step down and step up operation without an internal transformer and is ideal for motor control applications. Divan (1989) designed a resonant DC-link capable of bi-directional power flow. Tsai, Oruganti, and Lee (1989) investigated the control of a bi-directional parallel resonant converter. Tsai developed a technique of control based on the difference between the inverter switch angle and the output rectifier switch angle. This technique makes the performance of the converter independent of the load characteristics. Sul and Lipo (1990) proposed using a bi-directional parallel resonant converter for an induction motor drive. Chung, Shin, and Cho (1991) proposed a bi-directional series resonant inverter for an uninteruptible power supply. Kazimierczuk, Czarkowski, and Thirunaryan (1993) proposed a DC/DC converter consisting of two identical parallel resonant inverters that are phased shifted to produce the desired output voltage across the load.

TRANSIENT ANALYSIS OF THE CONVERTER CIRCUIT

A transient analysis of the shunt loaded series resonant converter will be presented in this paper. The Laplace transform method will produce closed form voltage and current equations that simplify the illustration of the circuit performance. The circuit can further be analyzed using Fourier series or state space averaging techniques for the complete steady state behavior.

A transient behavioral model of the converter will be developed in the discontinuous conduction mode with a duty cycle $k < 1$. The continuous mode is a degenerate case of the discontinuous conduction case where $k = 1$. The time domain solution from the inverse Laplace transform of the resulting equations will help set the values of capacitance, inductance, and switching frequencies that produce sufficient voltage gain and safe peak values of voltage and current for a base input and output voltage and power.

The proposed converter is shown below in Fig. 1 in both the two quadrant and four quadrant configurations. The design in this paper will cover the simpler two quadrant converter. The series resonant circuit is modified by placing the load across the inductor, creating a capacitor-inductor voltage divider. This technique takes advantage of the fundamental voltage-current relation of an inductor where,

$$V_L(t) = L \frac{dI_L(t)}{dt} \tag{1}$$

The impedance of the capacitor and inductor change with frequency, so the inductor current and voltage change also. This effect produces a voltage across the inductor that can be raised or lowered by changing the resonant excitation frequency. Because the load is connected across the inductor, the source to load voltage gain can be fixed by the inverter switching frequency, thereby eliminating the need for a transformer.

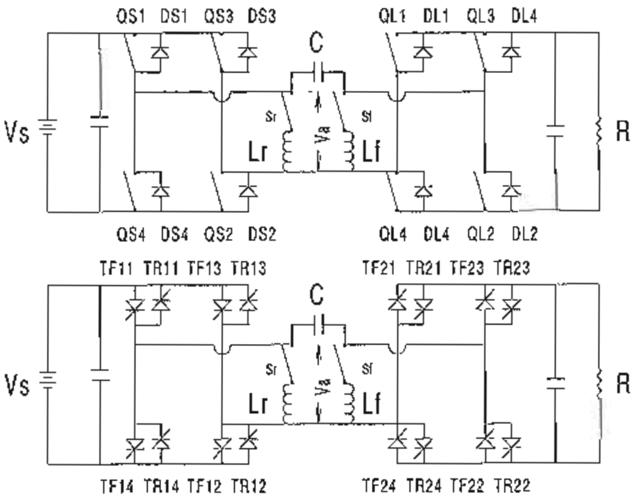

Fig. 1.
Two quadrant (upper) and four quadrant (lower) bi-directional shunt loaded series resonant converters

The converter in Fig. 1 operates in four states because of the four input voltage levels shown in Fig. 2. At t = t1 = 0 (state 1), QS1 and QS2 are on, so Va = Vs; at t = t2 = k ts/2 (state 2),. QS4 and QS2 are on, so Va = 0; at t = t3 = ts/2 (state 3), QS3 and QS4 are on, so Va = -Vs; and at t = t4 = (k+1) ts/2 (state 4), QS1 and QS3 are on, so Va = 0.

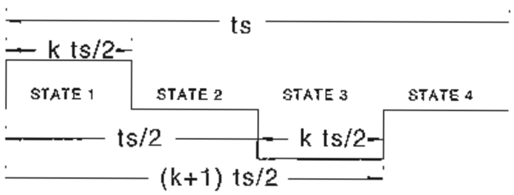

Figure 2.
Input Voltage Waveform for Duty Cycle k

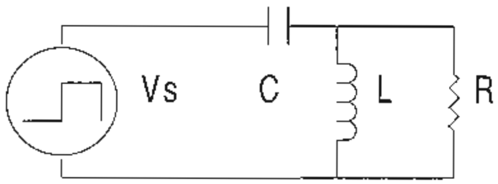

Figure 3.
Equivalent Circuit of Proposed Converter

Since the load is parallel to the resonant inductor (Fig. 3), load voltage is determined by the voltage across the inductor. The capacitor current, source voltage, capacitor voltage and inductor voltage have the following relations using Kirchoff's laws,

$$IC(t) = IL(t) + IR(t),\ Vs(t) = VC(t) + VL(t) \quad (2)$$

$$VC(t) = \frac{1}{C}\int IC(t)\,dt + VC(0),\ VL(t) = L\frac{dIL(t)}{dt} = R\,IR(t) \quad (3)$$

For Fig. 3, the following equations results when $V_C(t)$ and $V_L(t)$ are substituted into Vs(t).

$$Vs(t) = L\frac{dIL(t)}{dt} + \frac{1}{C}\int IL(t)dt + IR(t)\,dt + VC(0)$$

$$Vs(t) = L\frac{dIL(t)}{dt} + \frac{1}{C}\int IL(t)dt + \frac{1}{C}\int\frac{L}{R}\frac{dIL(t)}{dt}\,dt + VC(0)$$

$$Vs(t) = L\frac{dIL(t)}{dt} + \frac{1}{C}\int IL(t)\,dt + \frac{L}{RC}IL(t) + VC(0) \quad (4)$$

The Laplace transform Vs(t) is,

$$Vs(s) = sLIL(s) - LIL(0) + \frac{1}{sC}IL(s) + \frac{L}{RC}IL(s) + \frac{VC(0)}{s} \quad (5)$$

Gathering the terms for inductor current,

$$Vs(s) + LIL(0) - \frac{VC(0)}{s} = \left(sL + \frac{1}{C}\frac{L}{R} + \frac{1}{sC}\right)IL(s) \quad (6)$$

$$\frac{s}{L}\left(Vs(s) + LIL(0) - \frac{VC(0)}{s}\right) = \left(s^2 + s\frac{1}{RC} + \frac{1}{LC}\right)IL(s) \quad (7)$$

The Laplace transform of the inductor current in Fig. 3 is,

$$IL(s) = \frac{\frac{1}{L}(sVs(s) + sLIL(0) - VC(0))}{s^2 + s\frac{1}{RC} + \frac{1}{LC}} \quad (8)$$

We see the poles of $I_L(s)$ are solely determined by the equation,

$$s^2 + s\frac{1}{RC} + \frac{1}{LC} = \left(s + \frac{1}{sRC}\right)^2 + \left[\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2\right] \quad (9)$$

To insure stability, the circuit must be damped such that the load impedance is limited by,

$$\frac{1}{LC} > \left(\frac{1}{2RC}\right)^2, \qquad R > \frac{1}{2}\sqrt{\frac{L}{C}} \quad (10)$$

The resulting poles are,

$$s = -\alpha \pm j\omega r \quad (11)$$

where the natural resonant frequency is given by,

$$\omega r = \sqrt{\omega o^2 - \alpha^2},\ \omega o = \frac{1}{\sqrt{LC}},\ \alpha = \frac{1}{2RC} \quad (12)$$

We define the terms s1 and s2 such that,

$$s1 = -\alpha + j\omega r, \qquad s2 = -\alpha - j\omega r \quad (13)$$

where the sum and difference between s1 and s2 are,

$$s1 - s2 = j2\omega r,\ s2 - s1 = -j2\omega r,\ s1 + s2 = -2\alpha \quad (14)$$

Performing the inverse Laplace transform on $I_L(s)$, the inductor current becomes,

$$IL(t) = \frac{1}{L}\frac{s1\,Vs(s1) + s1LIL(0) - Vc(0)}{s1 - s2}e^{s1t} + \frac{1}{L}\frac{s2\,Vs(s2) + s2LIL(0) - Vc(0)}{s2 - s1}e^{s2t} \quad (15)$$

Using Equ. 1 to determine the voltage across the inductor,

$$V_L(t) = \frac{s1^2\ Vs(s1) + s1^2\ LI_L(0) - s1Vc(0)}{s1 - s2} e^{s1t} + \frac{s2^2\ Vs(s2) + s2^2\ LI_L(0) - s2Vc(0)}{s2 - s1} e^{s2t} \quad (16)$$

The input voltage consists of a series of step functions with initial and final voltage and current values that are the boundary conditions of the particular solutions. The transfer function of the input voltage step function is,

$$Vs(s) = \frac{Vs}{s} \quad (17)$$

For state 1 Vs(s) = +Vs/s, for state 2 Vs(s) = 0, for state 3 Vs(s) = -Vs/s, and for state 4 Vs(s) = 0, etc.. Therefore,

$$sVs(s) = \begin{matrix} +\ Vs\ state\ 1 \\ -\ Vs\ state\ 3 \\ 0\ \ state\ \ 2,4. \end{matrix} \quad (18)$$

$$s^2Vs(s) = \pm sVs\ or\ 0 \quad (19)$$

The inductor current in state 1 is,

$$I_L(t) = \frac{Vs}{\omega rL}\sqrt{A^2 + B^2}e^{-\alpha t}\sin(\omega rt + \beta) \quad (20)$$

where,

$$A = 1 - \frac{Vc(0)}{Vs} - \frac{\alpha LI_L(0)}{Vs}$$
$$B = \frac{\omega rLI_L(0)}{Vs}$$
$$\beta = \tan^{-1}\left(\frac{B}{A}\right) \quad (21)$$

The inductor voltage in state 1 is,

$$V_L(t) = Vs\frac{\omega o}{\omega r}\sqrt{A^2 + B^2}e^{-\alpha t}\cos(\omega rt + \beta + \phi) \quad (22)$$

where the angle of displacement ϕ is defined as,

$$\phi = \tan^{-1}\left(\frac{\alpha}{\omega r}\right) \quad (23)$$

The voltage across the inductor is the same as the unrectified output voltage and its magnitude is independent of the load impedance. The unrectified load current is given by,

$$I_R(t) = \frac{Vs}{R}\frac{\omega o}{\omega r}e^{-\alpha t}\sqrt{A^2 + B^2}\cos(\omega rt + \beta + \phi) \quad (24)$$

The capacitor current is determined by $I_C(t) = I_L(t) + I_R(t)$,

$$Ic(t) = IcMe^{-\alpha t}\sin(\omega rt + \beta + \theta) \quad (25)$$

where,

$$ICM = \frac{Vs}{R}\sqrt{A^2 + B^2}\sqrt{D^2 + 1}$$
$$D = \frac{R}{\omega rL} - \frac{\alpha}{\omega r} = \frac{R - \alpha L}{\omega r}$$
$$\sin(\theta) = \frac{1}{\sqrt{D^2 + 1}} \quad (26)$$

Finally, the capacitor voltage in state 1 is,

$$Vc(t) = \frac{ICM}{\omega oC}\begin{bmatrix}\cos(\beta + \theta + \phi) - \\ e^{-\alpha t}\cos(\omega rt - \beta - \theta - \phi)\end{bmatrix} + Vc(0) \quad (27)$$

For state 2, the input voltage Vs = 0 or sVs(s) = 0, so the inductor transfer function becomes,

$$I_L(s) = \frac{\frac{1}{L}(sLI_L(t1) - Vc(t1))}{s^2 + s\frac{1}{RC} + \frac{1}{LC}} \quad (28)$$

The solution for the inductor current in state 2 is then,

$$I_L(t) = I_L(t1)\sqrt{A2^2 + 1}e^{-\alpha(t-t1)}\sin(\omega r(t - t1) + \beta 2) \quad (29)$$

where,

$$A2 = -\frac{Vc(t1)}{\omega rLI_L(t1)} - \frac{\alpha}{\omega r}, \sin(\beta 2) = \frac{1}{\sqrt{A2^2 + 1}} \quad (30)$$

The inductor voltage (load voltage) in state 2 is,

$$V_L(t) = \omega oLI_L(t1)\sqrt{A2^2 + 1}\ e^{-\alpha(t-t1)}\cos(\omega r(t - t1) + \beta 2 + \phi) \quad (31)$$

The capacitor voltage will increase after many cycles, since a residual charge remains when the inverter current is switched off in each cycle, or $V_C(t1)_{state\ 1} = V_C(t1)_{state\ 2}$. Eventually, the initial capacitor voltage and the initial inductor current increase sufficiently (Equ. 26) to produce the desired voltage across the load. The capacitor voltage in state 2 becomes,

$$Vc(t) = Vc(t1)\frac{K}{Q}\sqrt{D^2+1} \left[\begin{matrix}\cos(\beta 2+\theta+\phi) - \\ e^{-\alpha(t-t1)}\cos(\omega(t-t1)-\beta 2-\theta-\phi)\end{matrix}\right] + Vc(t1) \quad (32)$$

where Q, the quality factor, is defined as $Q = \omega o/2\alpha$, D and θ are defined in Equ. 28, and,

$$K = \sqrt{\left(1+\frac{\alpha LIL(t1)}{Vc(t1)}\right)^2 + \left(\frac{\omega r LIL(t1)}{Vc(t1)}\right)^2} \quad (33)$$

Finally, the load current in state 2 is,

$$IR(t) = \frac{\omega o L}{R} IL(t1)\sqrt{A2^2+1}\; e^{-\alpha(t-t1)}\cos(\omega r(t-t1)+\beta 2+\phi) \quad (34)$$

The analysis of states 3 and 4 follow the same procedure as shown for states 1 and 2, except sVs(s) = -Vs in state 3.

The value for inductance L is determined from the resonant frequency and the resonant capacitance,

$$L = \frac{1}{\omega o^2 C} \quad (35)$$

The base load resistance R is directly determined from the relation of load voltage and load power,

$$R = \frac{Vo^2}{P} \quad (36)$$

where P is load power and Vo is the load voltage. The lower limit of resonant capacitance is determined by the damping condition $\omega o^2 > \alpha^2$, or $\omega o^2 > 1/(2RC)^2$, which implies,

$$C > \frac{P}{2\omega o Vo^2} \quad (37)$$

STEADY STATE SOURCE TO LOAD VOLTAGE GAIN

The gain of a circuit is a function used to understand the steady state or average behavior of the circuit. The effects of the exponential decay of the waveform are ignored, so α is neglected and only the periodic frequency ω is considered in the transfer functions. The source to load voltage gain is $GV(s) = V_R(s)/Vs(s)$, where $V_R(s) = s\,L\,I_L(s)$ and,

$$IL(s) = \frac{\frac{1}{L}(sVs(s)) + sLIL(0) - Vc(0))}{s^2 + s\frac{1}{RC} + \frac{1}{LC}} \quad (38)$$

The voltage gain $GV(s) = V_R(s)/Vs(s) = s\,L\,I_L(s)/Vs(s)$ is then,

$$GV(s) = \frac{\left(s^2 + s^2\frac{LIL(0)}{Vs(s)} - s\frac{Vc(0)}{Vs(s)}\right)}{s^2 + s\frac{1}{RC} + \frac{1}{LC}} \quad (39)$$

Since the quality factor $Q = \omega o/2\alpha = \omega oRC$, the source to voltage gain can be normalized using $\omega o^2 = 1/LC$,

$$GV\left(\frac{s}{\omega o}\right) = \frac{\left(\frac{s}{\omega o}^2 + \frac{s}{\omega o}^2\frac{LIL(0)}{Vs(s)} - \frac{s}{\omega o}\frac{1}{\omega o}\frac{Vc(0)}{Vs(s)}\right)}{\frac{s}{\omega o}^2 + \frac{s}{\omega o}\frac{1}{Q} + 1} \quad (40)$$

The Laplace transform for the full steady state waveform Vs(s) in the discontinuous mode (k < 1) in Fig. 2 is,

$$Vs(s) = \frac{Vs}{s}\left(1 - e^{-k\frac{ts}{2}s} - e^{-\frac{ts}{2}s} + e^{-(k+1)\frac{ts}{2}} + \ldots\right) \quad (41)$$

$$sVs(s) = Vs\left(\frac{1-e^{-k\frac{ts}{2}s}}{1+e^{-\frac{ts}{2}s}}\right) \quad (42)$$

The transfer function sVs(s) can be written in hyperbolic functions,

$$sVs(s) = Vs\frac{\sinh\left(k\frac{ts}{4}s\right)}{\cosh\left(\frac{ts}{4}s\right)} \quad (43)$$

Substituting $s = j\omega s$ in Vs(s) to see the effect of switching frequency,

$$Vs(j\omega s) = \frac{1}{j\omega s}Vs\frac{\sinh\left(k\frac{ts}{4}j\omega s\right)}{\cosh\left(\frac{ts}{4}j\omega s\right)} \quad (44)$$

The term $\cosh(ts/4\, j\omega s) = \cos(tsfs\pi/2) = 0$, which simplifies the gain to the expression,

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Since 1985, he has worked as an electrical engineer for the Aero Propulsion and Power Directorate, of Wright Laboratory at Wright-Patterson Air Force Base, Ohio studying and developing power converters, power conditioners, power distribution systems, and pulse power systems for aircraft, spacecraft, and electric vehicles. From 1983 to 1985, he worked as an electronics engineer in the Avionics Laboratory, Air Force Wright Aeronautical Laboratories at Wright-Patterson Air Force Base, Ohio studying and developing gas lasers. He is a member of the IEEE Power Electronics Society.